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ON THE RESPONSE OF AIRPLANES IN A THREE-DIMENSIONAL GUST FIELD

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The response of an airplane in a 3-dimensional gust field is examined with the intent of reducing the problem to the simplest terms. It is shown that response evaluation may be reduced to separate treatment of the longitudinal and lateral response cases. The longitudinal case reduces to considering the vertical gust velocities only, and generally the degrees of freedom of vertical motion, pitch, and vertical bending modes; vertical motion is the prime degree of freedom. The lateral case reduces to side gust velocities only, with degrees of freedom of side motion, yaw, and side bending; yaw is the prime degree-of-freedom in this case. The longitudinal gusts are found to be unimportant. The analysis shows that the results from the separate longitudinal and lateral response evaluations must be combined to obtain the fuselage loads and loads on T-type tails.

SYMBOLS

a	slope of the lift curve
a_H	slope of the lift curve associated with horizontal tail
a_V	slope of the lift curve associated with vertical tail
b	span
c	chord
c_o	mean aerodynamic chord
e_H, e_V	moment arms
$H(\omega)$	frequency response functions
L	lift, also turbulence scale
s	separation distance
S	wing area

S_H, S_V	areas associated with horizontal and vertical tails
t	time
u	longitudinal component of turbulence
U	airplane velocity
v	lateral component of turbulence
w	vertical component of turbulence
W	airplane weight
x,y,z	coordinate axes system; also displacements
α	angle of attack
Γ	dihedral angle
θ	pitch angle
ρ	air density
σ_w	rms value for vertical gusts w
ϕ	roll angle
ϕ_w	power spectrum of vertical gust velocities
ψ	yaw angle
ω	circular frequency
Ω	spatial frequency, $\frac{\omega}{U}$

INTRODUCTION

This report deals with the response of an airplane in a 3-dimensional gust field. Various aspects of the problem have been considered in previous studies, references 1-17, but coverage relative to the 1-dimensional response case (vertical gusts random in the flight direction only) is small, particularly with respect to practical applications. The studies that have been made give consideration to such items as the derivation of cross-spectra for isotropic turbulence, the effect of considering the gusts to be random in the spanwise direction, as well as the direction of flight (2-d gust field), longitudinal and lateral response, and horizontal and vertical tail loads.

The main purpose of this report is to give a general assessment of the 3-d turbulence encounter problem with the aim of reducing response considerations to the simplest terms. The relative magnitudes of the various forcing terms due to each of the three gust components, u , v , and w , are developed. The terms that are felt to be of significance are singled out. Reduction to the subcases of longitudinal and lateral response is made, but the essential aspects of 3-d encounter are retained. The means for treating tail and fuselage loads due to the combined action of vertical and horizontal gusts is developed, with particular attention being given to T-tails, where a superposition of vertical loadings due to both side and vertical gusts is involved.

THE 3-D GUST FIELD

Figure 1 depicts the three-dimensional gust field that is considered in this analysis. The longitudinal component u is

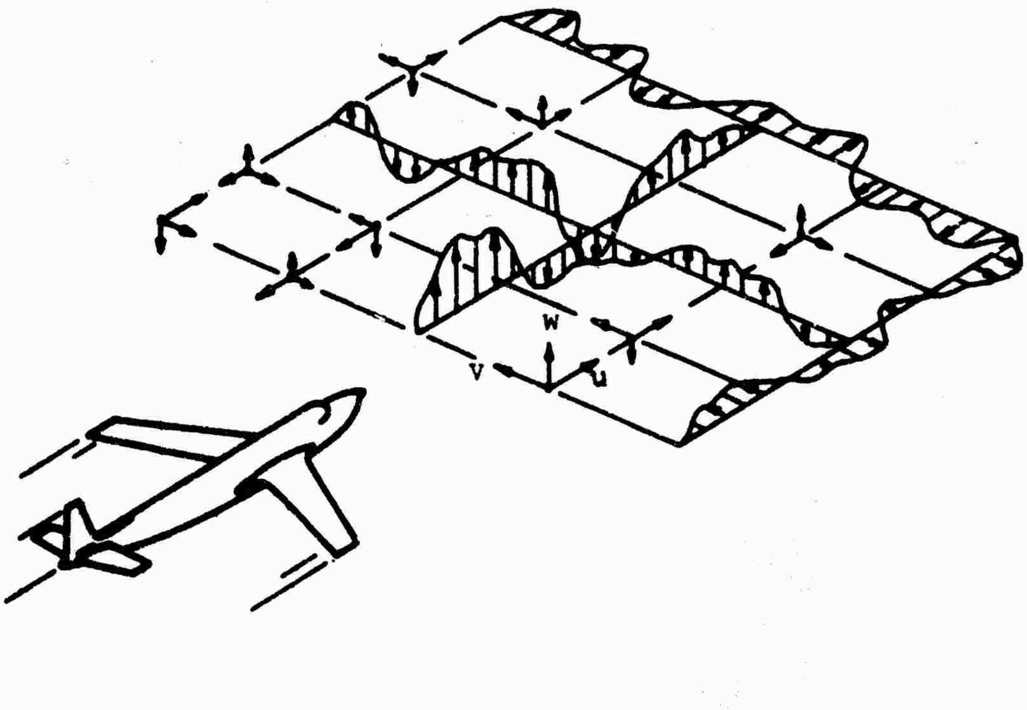


Figure 1. Three-dimensional random gust encounter

considered to be random in the direction of flight and in the span-wise direction. This component, though included at the start, is shown to be of negligible importance in response treatments. The side gusts v are considered to be random in the flight direction, but due to the smallness of the airplane in depth, are considered uniform in the vertical direction. The vertical component w is considered random in both the flight direction and in the spanwise direction. Subsequently, v and w components are shown to be the prime components that must be included in response evaluation.

The gust field is considered to be isotropic. Various studies show that the spectral relations due to von Kármán apply quite well to atmospheric turbulence, and hence they are adopted as the basis for the mathematical modelling of the turbulence field. The relations are (references 18 and 19):

for w and v,

$$\phi_w(\Omega L) = \frac{\sigma_w^2}{\pi} \frac{1 + \frac{8}{3}(1.339\Omega L)^2}{\left[1 + (1.339\Omega L)^2\right]^{11/6}} \tag{1}$$

for u,

$$\phi_u(\Omega L) = \frac{2\sigma_u^2}{\pi} \frac{1}{\left[1 + (1.339\Omega L)^2\right]^{5/6}} \tag{2}$$

Isotropy implies that the severity of all three components is equal; thus, as expressed by the rms values

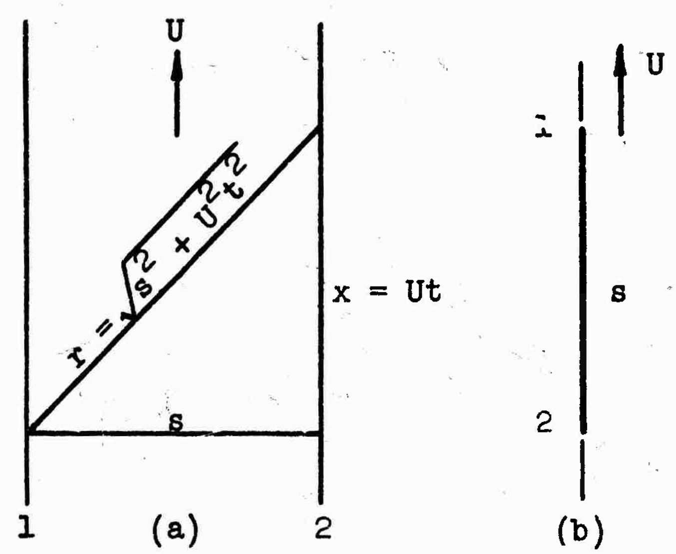
$$\sigma_u = \sigma_v = \sigma_w \tag{3}$$

Isotropy also means that the cross-correlation and cross-spectra between the three components vanish; thus

$$\phi_{uv} = \phi_{uw} = \phi_{vw} = 0 \tag{4}$$

These relations have significant consequence, since response evaluations are thereby greatly simplified.

In addition, the condition of isotropy allows for the convenient evaluation of the cross-spectra for the individual gust components. Two of the more important cross-spectra cases are depicted in the following sketches



In case (a) we are interested in the cross-spectra between the w values along path 1 and the w values along path 2, as would be involved in the consideration of nonuniform spanwise gusts. Because of isotropy, we can derive this function directly from the point correlation function $R(x)$. We replace x by r to obtain the cross-correlation function

$$R_{12}(x,s) = R\left(\sqrt{s^2 + x^2}\right)$$

The cross-spectra follows as the Fourier transform of this function. Reference 16 applies this technique to derive the cross-spectral functions associated with equations (1) and (2).

In case (b) we are interested in the cross-spectra between the w values at point 1 and the w values at point 2, as would be involved in considering the gust loading effects on the wing and horizontal tail. In this case, it is common to consider that the w values at point 2 are simply the w values that were sensed at point 1, a time Δt earlier, where Δt is the time required for a point on the airplane to travel the distance s between the points, or $\Delta t = \frac{s}{U}$. (This is the temporarily frozen gust field concept.) In terms of the point correlation function, the cross-correlation function for this case is simply

$$R_{12}(t,s) = R(t - \frac{s}{U})$$

From this relation the cross-spectra for case (b) follows as

$$\phi_{12}(\omega,s) = e^{-i\frac{\omega s}{U}} \phi(\omega)$$

where $\phi(\omega)$ is simply the point spectrum, equation (1).

Application of cross-spectra in multidimensional response treatment can be made by the general methods outlined in reference 1.

GENERAL RESPONSE CONSIDERATIONS

In multidimensional response studies wherein the airplane is treated as a rigid body, a stability-axes system has often been used, so that direct use of stability derivatives could be made. We shall, however, speak mostly in terms of a rectangular coordinate system x-y-z moving with the airplane velocity U, but otherwise inertially fixed; x is the flight direction, y the spanwise direction, and z is vertical. We assume that if information on the stability derivatives of the airplane is available, it can be incorporated by a suitable axes transformation.

In general, the airplane is considered to respond in three basic ways

a) translation in the x, y, and z directions

b) rotation about the x, y, and z axes

c) flexible body deformations.

As a way of discussing the response problem in general, we speak in terms of the basic normal mode approach. Thus, the following equation applies

$$M_n \ddot{a}_n + \beta_n \dot{a}_n + \omega_n^2 M_n a_n = \int p z_n dS \tag{5}$$

This equation applies, of course, to all six rigid-body modes, as well as any flexible mode that is included. The three gust components enter through the manner in which they give rise to the pressure term p that appears in the integral. This pressure term also includes all forces that develop as a result of aircraft motion. Through use of equation (5), general response treatments may be made, such as given in reference 1.

RELATIVE SIZE OF FORCING FUNCTIONS

In this section we consider the application of equation (5) to all the rigid-body modes, and establish how each of the three gust components, u , v , and w , influence the magnitude of the forcing term that appears on the right-hand side.

To give a quick overall perspective of the relative magnitude of the various forcing terms, we give at the onset a summary of the results of this section, Table I. This table indicates the order of magnitude of the forces and moments that are applied to the aircraft due to each of the three gust components; W is aircraft weight, b the span, α is the angle of attack necessary to sustain level flight of the airplane at the speed U . In the development of the table it was assumed that both the left and right halves of the wing experience different gust velocities, and thus the effects of spanwise variations in turbulence are also brought out. Example interpretations are as follows. The magnitude of the force in the z direction due to u is seen to be essentially the weight times a characteristically small angle; if u_1 and u_2 have rms values of 3 fps, and U is 500 fps, then a rms force in the z-direction due to u of only about .01 W is indicated. By contrast consider the z-force due to w ; we see the force is given by the weight times the ratio of comparable angle of attacks. Thus the z-force due to w is on the order of the weight of the airplane. Throughout the derivations, the weight of the airplane expressed in terms of the steady-state lift is used as a convenient reference force magnitude; hence, the expression

$$L = W = \frac{a}{2} \rho U^2 S \alpha \tag{6}$$

was employed where a is slope of the lift curve, S is wing area and α is the steady-state angle of attack for level flight. Since we seek only to establish the magnitude of the forces and moments that act on the airplane, without being precise in detail, we use an analysis akin to strip-theory, and assume that the gusts act in quasi-steady sense only.

Force in the x-direction.- It is usually assumed that the gust components u , v , and w cause negligible perturbations in the x-direction, and this assumption is also adopted here. The essential reasoning behind this quite plausible assumption is that the airplane in the x-direction is a purposely tailored streamlined shape. Of the three components, u causes the largest variation in the x-force, or drag, but the force is small as the following analysis shows. Consider the drag of the airplane expressed as

$$D = D_1 U^2$$

With the use of differential calculus techniques, incremental changes in drag due to changes in U are given by

$$\Delta D = 2D_1 U \Delta U$$

If we combine these two equations, and introduce the fact that $L = W$, we find

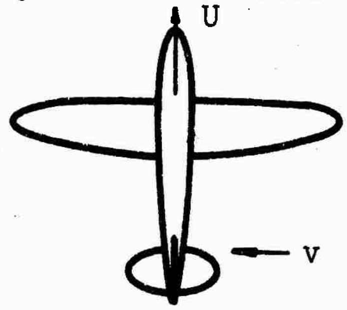
$$\frac{\Delta D}{W} = \frac{D}{L} \frac{2\Delta U}{U}$$

If ΔU is regarded as the gust component u, then we see that only small drag variations in terms of airplane weight are involved; for example, a rms value of $u = 3$ fps, $U = 500$ fps, $\frac{L}{D} = 8$, indicates

$$\Delta D = .0015\ W$$

Since this drag variation is small, and since the x-force variations due to v and w are even smaller, we arrive at the conclusion that we can ignore the x-force due to all three components, as the table indicates.

Force in the y-direction.- No fundamental mechanism exists for producing force in the y-direction due to u and w; therefore, a blank spot for both of these terms is shown in the table. The v component can produce a small side force by acting on the fuselage and vertical tail. With reference to the following sketch



the side force due to v is given by

$$F_{y_v} = \frac{a_v}{2} \rho U^2 S_v \frac{v}{U}$$

where a_v and S_v are associated with an equivalent vertical tail which produces the same side force as does the actual fuselage and tail combination of the airplane when in an angle of yaw equal to $\frac{v}{U}$. With the use of equation (6), this equation becomes

$$F_{y_v} = \frac{a_v}{a} \frac{S_v}{S} \frac{1}{\alpha} \frac{v}{U} W \tag{7}$$

This relation shows that the side force due to v is small; for example, with $\frac{a_v}{a} = \frac{1}{2}$, $\frac{S_v}{S} = \frac{1}{7}$, and even with $\frac{v}{U} = \alpha$, a side force equal to 1/14 the weight is shown. We suggest later the omission of this force in first-order response evaluations.

Force in the z-direction.- The force in the z-direction due to u is derived as follows. The lift on the wing is given by (with the use of strip theory)

$$L = \frac{a}{2} \rho \int_{-b/2}^{b/2} c u_1^2 \alpha dy$$

where u_1 is the instantaneous flow velocity over a chord due to the steady forward velocity U and the longitudinal gusts u; that is

$$u_1 = U + u$$

If this expression is substituted in the integral, then the change in L from the steady-state value is found to be (ignoring the u^2 term)

$$F_{z_u} = \frac{a}{2} \rho \int_{-b/2}^{b/2} c 2Uu\alpha dy$$

With equation (6), this equation may be written

$$F_{z_u} = W \int_{-1}^{1} \frac{c}{c_o} \frac{u}{U} d\left(\frac{y}{b/2}\right) \tag{8}$$

where c_o is the mean aerodynamic chord. To bring out the results a little more clearly we apply this equation to the idealized wing shown in figure 2. In this figure, we consider u_1 to be the u

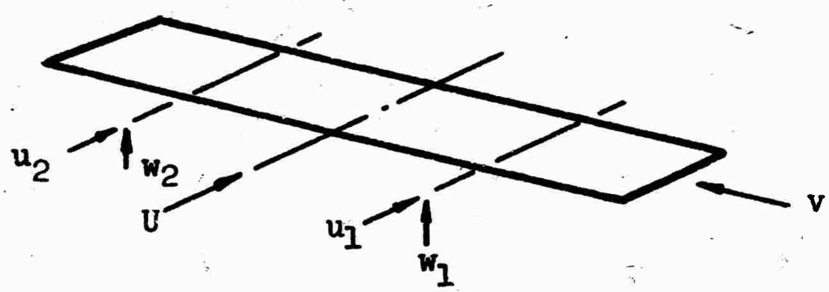


Figure 2. Wing system used to derive gust forcing functions

velocity that acts over the left half of the wing, and u_2 the u velocity that acts over the right half. For this case, equation (8) yields

$$F_{z_u} = W\left(\frac{u_1}{U} + \frac{u_2}{U}\right) \tag{9}$$

which is the value shown in the table. The F_z force due to u is thus seen to be a small component of the weight, roughly the weight times a small angle (an angle of attack given by $2\frac{u}{U}$) . We will eventually neglect this force.

The gust component v produces no force in the z-direction and so no entry in the table is made.

The gust component w is the strongest F_z producer. The equation for lift produced by w is given as

$$L = F_{z_w} = \frac{a}{2}\rho\int_{-b/2}^{b/2} cU^2\,\frac{w}{U}\,dy$$

With equation (6) this equation becomes

$$F_{z_w} = \frac{W}{\alpha}\,\frac{1}{2}\int_{-1}^{1}\frac{c}{c_o}\;\frac{w}{U}\,d\left(\frac{y}{b/2}\right) \tag{10}$$

In application to our simple model wing of figure 2, this equation yields

$$F_{z_w} = \frac{W}{2}\frac{1}{\alpha}\left(\frac{w_1}{U} + \frac{w_2}{U}\right) \tag{11}$$

which is the result given in Table I. Notice the magnitude of this term by comparison with the F_z term for u; the value is essentially the F_z term for u divided by a small angle, and so a force an order of magnitude greater is found. Since the force is equal to the weight multiplied by the ratio of angles of attack, which are roughly equal to one another, the force is essentially the weight of the airplane. The F_z due to w is thus significant in response evaluations.

Rolling moment.- We consider the rolling moments due to u and w first. These rolling moment forcing terms come from the same lifting forces as are involved in deriving equations (8) and (10). For roll, a moment arm y would also appear in the integrals; thus, the rolling moments may be shown to be

$$M_{R_u} = W\frac{b}{2}\int_{-1}^{1}\frac{c}{c_o}\frac{u}{U}\frac{y}{b/2}\,d\left(\frac{y}{b/2}\right) \tag{12}$$

$$M_{R_w} = \frac{W}{\alpha}\frac{b}{4}\int_{-1}^{1}\frac{c}{c_o}\frac{w}{U}\frac{y}{b/2}\,d\left(\frac{y}{b/2}\right) \tag{13}$$

These equations indicate that the forcing moments from the left and right wings try to cancel one another. Indeed, if the gusts u and w were uniform in the spanwise direction, no rolling power would be created. For u and w variable in the spanwise direction, some rolling moment can develop. The application of equations (12) and (13) to the example wing of figure 2 gives the results

$$M_{R_u} = \frac{Wb}{4}\left(\frac{u_1}{U} - \frac{u_2}{U}\right) \tag{14}$$

$$M_{R_w} = \frac{Wb}{8}\frac{1}{\alpha}\left(\frac{w_1}{U} - \frac{w_2}{U}\right) \tag{15}$$

which are the results given in Table I. We hold back further discussion of the magnitudes of these terms until the rolling moment due to v is established.

The forcing rolling moment due to v is associated with a dihedral effect. The local angle of attack due to a side velocity v can be shown to be

$$\alpha = \Gamma \frac{v}{U}$$

where Γ is the dihedral angle. Since the dihedral angles of the right and left wings are opposite in sign, the local angle of attack and, hence, the lift will be opposite also. Additive rolling moments due to the right and left halves are therefore produced. The rolling moment due to v is

$$M_{R_v} = \frac{a}{2} \rho \, 2 \int_0^{b/2} cU^2 \Gamma \frac{v}{U} y \, dy$$

which by equation (6) may be expressed

$$M_{R_v} = W \frac{\Gamma}{\alpha} \frac{b}{2} \int_0^1 \frac{c}{c_o} \frac{y}{b/2} \frac{v}{U} d\left(\frac{y}{b/2}\right) \tag{16}$$

If v is assumed constant across the span, the application of this equation to the example wing of figure 2 gives

$$M_{R_v} = \frac{Wb}{4} \frac{\Gamma}{\alpha} \frac{v}{U} \tag{17}$$

as is shown in Table I.

We now compare the magnitudes of the three terms. The factor $\frac{b}{4}$ can be considered as a common moment arm; thus we can compare the force producing the rolling moments. For M_{R_u} we see the force is essentially the weight W times the difference of small angles; in general, therefore, only a small force is involved. We can thus infer that this rolling moment term can be dropped, and later we will do so. The M_{R_w} term is seen to be essentially the M_{R_u} term divided by α ; thus, even though the difference of small angles is involved, the term is an order of magnitude greater than the M_{R_u} term. It is therefore usually large enough so that it should be considered. For M_{R_v} , the factor Γ essentially takes the place of the difference of small angles; because the term involves division by α , however, just as with the M_{R_w} term, it can have a magnitude as great the M_{R_w} term. The M_{R_v} term must therefore generally be retained.

Pitching moment.- Because of lift variations similar to those described under the F_z term due to u , a small pitching moment due to u may occur; the moment is quite small, however, and may be neglected, as the table shows. There is no pitching moment due to v , and so there is no entry for this term either. The main pitching moment is caused by w . For this treatment we assume that the moment is primarily due to a vertical tail force that develops on the horizontal tail and which acts at some moment arm e_H from the c.g. This moment is given by

$$M_{P_w} = \frac{a_H}{2} \rho S_H U^2 \frac{w}{U} e_H$$

By equation (6) this value becomes

$$M_{P_w} = W \frac{a_H}{a} \frac{S_H}{S} \frac{1}{\alpha} \frac{w}{U} e_H \tag{18}$$

which is the result shown in the table. If we consider that α is roughly equal to $\frac{w}{U}$, this equation shows that the forcing pitching moment is a good percentage of the weight W times a substantial moment arm e_H . Thus, the pitching moment due to w may, in general, be of concern.

Yawing moment.- The yawing moments are analogous to the pitching moments. The yawing moment due to u is negligible, and w does not create any. The yawing moment due to v is similar in development to the pitching moment due to w . Essentially, we consider the side force F_y due to v that develops on the vertical tail, and assume it to act through a moment arm about the c.g. of e_V . The result is

$$M_{Y_v} = W \frac{a_V}{a} \frac{S_V}{S} \frac{1}{\alpha} \frac{v}{U} e_V \tag{19}$$

as is shown in the table. This moment is found to be of significance in lateral response considerations.

Vertical motion modes.- As indicated in Table I, only w has practical significance in exciting the normal modes of the airplane which are characterized primarily by vertical motion. The extent of excitation of a normal mode is governed by a generalized force (right side of equation (5)) which depends on the modal shape being excited. Since this exciting force varies with each modal shape, no magnitude evaluations are made for entry into the table.

Side bending modes.- For the modes which are characterized mainly by side bending motion of the fuselage, only v is significant as a forcing function. We indicate this fact by a check mark in Table I, analogous to the check mark under w for the vertical motion modes.

SEPARATION OF LONGITUDINAL AND LATERAL RESPONSE

When we examine Table I, we note that an orderly grouping of terms appear, and recognize that the longitudinal and lateral response cases may be separated, thus leading to significant simplifications in 3-d response evaluations. Before discussing this separation, we make some observations of interest about Table I. We note that of all the degrees of freedom, only roll is influenced directly by all three gust components. In spite of this fact, we will later suppress roll as a degree of freedom, since it does not have a great influence on load response evaluations. The previous section indicated that the terms under u are small and so we will now drop them. Reference 6 verifies, in fact, that the u forcing term for roll has very little effect on the roll response. With the elimination of the x degree of freedom, and the u forcing terms, the general rigid-body response equations would reduce to the form

$$\left[\quad C \quad \right] \left| \begin{matrix} y \\ z \\ \phi \\ \theta \\ \psi \end{matrix} \right| = \left| \begin{matrix} F_{y_v} \\ F_{z_w} \\ M_{r_v} + M_{r_w} \\ M_{P_w} \\ M_{Y_v} \end{matrix} \right| \qquad (20)$$

In this symbolic representation, the matrix C is considered to include the inertia terms, cross coupling, and all force and moment terms that are associated with airplane motion.

If the angular motions are small, then the longitudinal and lateral motions become essentially uncoupled and we may split equation (20) into two separate smaller sets, one for longitudinal response, one for lateral; we discuss these sets separately.

Longitudinal response.- If we extract the longitudinal response equations from equation (20) and introduce the flexible response modes, which were temporarily suppressed, we find the following set applying to longitudinal response

$$\begin{bmatrix} \\ & & \\ & A & \\ & & \\ \end{bmatrix} \begin{vmatrix} z \\ \theta \\ \cdot \\ a_n \\ \cdot \end{vmatrix} = \begin{vmatrix} F_{z_w} \\ M_{P_w} \\ \cdot \\ Q_{n_w} \\ \cdot \end{vmatrix} \tag{21}$$

We notice that only w is involved. Generally, the equation applies to a w-gust field which is both random in the direction of flight and the spanwise direction. The matrix A contains inertia terms and aerodynamic terms, associated with airplane motion, as found by lifting surfaces methods, for example. Various simplifications involve considering the gusts to be uniform spanwise, and of suppressing various degrees of freedom.

Of all the degrees of freedom in equation (21), z, the rigid-body vertical translation, is the most significant. Consideration of z alone, as is often done, leads to first order results for loads; the equation for this case appears simply as

$$A_1 z = F_{z_w} \tag{22}$$

In spite of the fact that we refer to this equation as a rigid-body equation, it should be noted that if this equation is used for airplanes with swept wings, care should be taken to use the slope of the lift curve for the flexible airplane so that steady state wing bending effects are taken into account.

Lateral response.- The lateral response equations that are indicated by equation (20), together with an appropriate flexible mode response, are

$$\begin{bmatrix} & & \\ & B & \\ & & \end{bmatrix} \begin{vmatrix} y \\ \phi \\ \psi \\ b_1 \end{vmatrix} = \begin{vmatrix} F_{y_v} \\ M_{R_v} + M_{R_w} \\ M_{Y_v} \\ Q_{1_v} \end{vmatrix} \tag{23}$$

We notice that the lateral response is, in general, influenced by both v and w . In equation (23) we indicate only one flexible mode, since it is felt that the use of one should be adequate for most all practical applications.

If motion response of the airplane is of prime consideration, then the degree of freedom of roll (ϕ) should be included in applications, so as to adequately represent the dutch roll mode of the airplane. The roll freedom may be suppressed, however, if tail and fuselage loads, rather than overall airplane motion, are of chief concern, as is usually the case; these loads are not affected greatly by roll motion. Yawing motion appears to be the biggest factor associated with the loads induced. The lateral response case treated in reference 20 was in fact examined on this point. The axes systems were first transformed from the stability system to the system of this paper, using the transformation $v = \dot{y} - U\psi$. It was found that the results obtained for the case of $\dot{y} = 0$, using only ψ , were essentially the same as given in the reference. With ϕ suppressed, the lateral response equations become

$$\begin{bmatrix} & & \\ & B_1 & \\ & & \end{bmatrix} \begin{vmatrix} y \\ \psi \\ b_1 \end{vmatrix} = \begin{vmatrix} F_{y_v} \\ M_{Y_v} \\ Q_{1_v} \end{vmatrix} \tag{24}$$

We note that only v is now involved.

The most significant degree of freedom for lateral response appears to be the rigid-body yaw (ψ); the equation for this case is simply

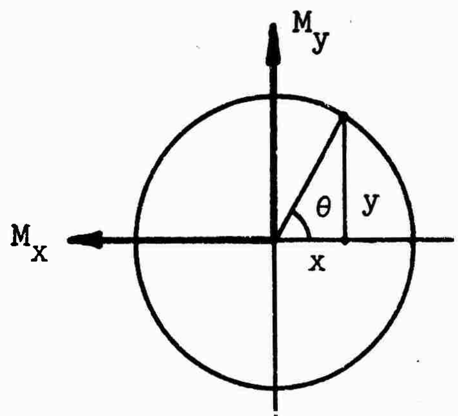
$$B_2\psi = M_{Y_v} \tag{25}$$

Use of this equation should give good first order results for lateral loads. We wish to emphasize the contrasting significance of equations (22) and (25). For longitudinal response, rigid-body vertical motion is predominant; this result may be attributed to the fact that large aerodynamic surfaces are exposed to the gusts w. For lateral response, rigid-body yaw is the most significant; in this case the amount of aerodynamic surfaces exposed to v is small, but the surfaces that are involved can create powerful yawing moments.

FUSELAGE LOADS

We have seen that the 3-d gust encounter problem may be separated into longitudinal and lateral response considerations. The wing loads, for design purposes, are established by the longitudinal response analysis, using equations (21) or (22). The fuselage loads should, however, be established by combining the longitudinal and lateral responses. Even though the responses are established in an uncoupled sense, a combining of loads is necessary to arrive at design loads. The following analysis illustrates the procedure.

Consider a fuselage cross section and the moments that act at this cross section, as shown in the following sketch



The moment M_x is considered to be the result of encountering the gusts w ; M_y is due to v . These moments may be given as

$$M_x = \int_{-\infty}^{t} w(\tau)h_x(t - \tau)\ d\tau \tag{26}$$

$$M_y = \int_{-\infty}^{t} v(\tau)h_y(t - \tau)\ d\tau \tag{27}$$

where h_x is the x-moment that develops due to a unit-impulse w gust, and h_y is the y-moment that develops due to a unit-impulse v gust. (These impulse functions are referred to here in implicit form only, since they are really not derived.)

The stress at point A is given by

$$s = \frac{M_x y}{I_x} + \frac{M_y x}{I_y}$$

If we substitute equations (26) and (27) into this expression, and take the Fourier transform, we obtain

$$F_s = \frac{y}{I_x} H_x F_w + \frac{x}{I_y} H_y F_v$$

where H_x and H_y are the frequency response functions for M_x and M_y due, respectively, to unit sinusoidal w and v gusts, and F_w and F_v are the Fourier transforms of the w and v gusts; the H_x and H_y functions are the important response functions that require evaluation. The spectrum for s follows from this equation as

$$\phi_s(\omega) = \frac{y^2}{I_x^2} |H_x|^2 \phi_w + \frac{x^2}{I_y^2} |H_y|^2 \phi_v \tag{28}$$

In the derivation of this equation, use was made of the fact that the cross-spectrum between v and w vanishes, equation (4). Since $\phi_v = \phi_w$, equation (28) may be written

$$\phi_s(\omega) = |H|^2 \phi_w \tag{29}$$

where

$$|H|^2 = \frac{y^2}{I_x^2} |H_x|^2 + \frac{x^2}{I_y^2} |H_y|^2$$

The rms value σ_s of stress, and N_o value, for use in design, follow directly from equation (29) in the customary fashion.

It is of interest to note how the spectral results for stress differ from the result that is obtained when M_x and M_y are static-type moments. Consider the cross section to be circular and let

$$x = r \cos \theta$$

$$y = r \sin \theta$$

Then, if the moments are static, the stress is given by

$$s = \frac{M_x r}{I_x} \sin \theta + \frac{M_y r}{I_y} \cos \theta$$

The angle leading to maximum stress is given by

$$\frac{ds}{d\theta} = \frac{M_x r}{I_x} \cos \theta - \frac{M_y r}{I_y} \sin \theta = 0$$

We solve this equation for θ, and substitute the result in the equation for s to find the maximum stress. The point is, when static moments are involved, there is a position θ which leads to maximum stress, as might be expected.

By contrast, the spectral result for stress for this circular cross-section case, as found through equation (28), is of the form

$$\sigma_s^2 = \sigma_1^2 \sin^2 \theta + \sigma_2^2 \cos^2 \theta$$

To find the location of maximum σ_s we differentiate and obtain

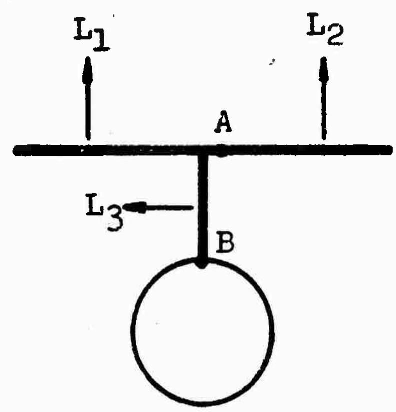
$$\frac{d\sigma_s^2}{d\theta} = 2(\sigma_1^2 - \sigma_2^2) \sin \theta \cos \theta = 0$$

From this expression we see that maximum σ_s is either at $\theta = 0$ or $\theta = \frac{\pi}{2}$. An exception occurs when $\sigma_1 = \sigma_2$; in this case, σ_s is invariant with respect to θ.

TAIL LOADS

For 3-d gust response, tail loads merit a special treatment. For conventional tails, no unusual problem arises. The longitudinal response involving w is used to calculate the loads on the horizontal tail. In similar fashion, the lateral response involving v is used for the vertical tail loads. For T-tails, load determination is more involved and requires the combining of loads due to the longitudinal and lateral response. The following analysis depicts the situation.

Consider a T-tail as shown in the following sketch



and consider the tail to have a pitch attitude θ and a yaw attitude ψ. The use of aerodynamic interference theory indicates that the loads shown are given by equations of the type

$$L_1 = a_1\theta - b_1\psi$$

$$L_2 = a_1\theta + b_1\psi \tag{30}$$

$$L_3 = c_1\psi$$

Thus, a yawing motion creates loads on the horizontal tail, as does motion in pitch. It should be mentioned that θ may arise from at least three sources, an actual pitch position θ, an effective angle of attack $\frac{\dot{z}}{U}$ due to vertical motion, and a gust angle of attack $\frac{w}{U}$; similar comments apply to ψ.

By equation (30), we see that the loads on the horizontal tail are due to both the longitudinal and lateral responses. A combining of loads must therefore be considered; this combining process is similar to that given in the previous section on fuselage loads. We illustrate it here in connection with determining the bending moment at point A shown in the sketch. The moment is given by

$$M_A = \int_{-\infty}^{t} w(\tau)h_w(t - \tau)d\tau + \int_{-\infty}^{t} v(\tau)h_v(t - \tau)\, d\tau$$

Reduction of this equation to spectral form gives the result

$$\phi_{M_A} = |H_w|^2 \phi_w + |H_v|^2 \phi_v$$

where H_w and H_v are the frequency response functions for moment at A due to unit sinusoidal w and v gusts, respectively. In the derivation of this equation use was again made of the fact that $\phi_{vw} = 0$, equation (4). Since $\phi_v = \phi_w$, the equation becomes

$$\phi_{M_A} = \left(|H_w|^2 + |H_v|^2\right)\phi_w \tag{31}$$

Design rms values for bending moment, and associated N_o, follow from this equation.

The establishment of the moment at point B does not involve w. Care must be taken, however, to include the load L_3, and the loads L_1 and L_2, when deriving this moment; in this instance, lateral response alone is involved.

CONCLUDING REMARKS

This report considers the 3-dimensional gust response problem of an airplane. The aim was mainly to suggest ideas on how the problem could be reduced to simplest terms. Table II is presented by way of summarizing the basic notions of the paper. Some of the chief points are as follows:

a) The x degree of freedom can be ignored.

b) The longitudinal gusts u may be ignored.

c) Longitudinal and lateral motion may be separated and treated in an uncoupled way.

d) Only the vertical component w enters into the longitudinal response problem.

e) The complete lateral response which includes the dutch roll mode involves both v and w.

f) For loads determination in the lateral response case, roll may be suppressed, and hence treatment is in terms of v only.

g) The longitudinal response determines the wing loads for use in design; for the very large aircraft, consideration should be given to the spanwise variations in gusts, especially if the turbulence scale is found to be less than 1000 ft.

h) Tail and fuselage loads are found by combining the results that are obtained from separate longitudinal and lateral response evaluations.

i) First-order load results can be obtained by considering rigid-body vertical motion only in connection with w, and rigid-body yawing motion only in connection with v.

TABLE I

MAGNITUDE OF FORCING TERMS DUE TO 3-D GUSTS

	u	v	w
x - Force	0	0	0
y - Force	0	$W \frac{a_v}{a} \frac{S_v}{S} \frac{1}{\alpha} \frac{v}{U}$	0
z - Force	$W\left(\frac{u_1}{U} + \frac{u_2}{U}\right)$	0	$\frac{W}{2} \frac{1}{\alpha}\left(\frac{w_1}{U} + \frac{w_2}{U}\right)$
Rolling Moment M_R	$\frac{Wb}{4}\left(\frac{u_1}{U} - \frac{u_2}{U}\right)$	$\frac{Wb}{4} \frac{\Gamma}{\alpha} \frac{v}{U}$	$\frac{Wb}{8} \frac{1}{\alpha}\left(\frac{w_1}{U} - \frac{w_2}{U}\right)$
Pitching Moment M_P	0	0	$W \frac{a_H}{a} \frac{S_H}{S} \frac{1}{\alpha} \frac{w}{U} e_H$
Yawing Moment M_Y	0	$W \frac{a_v}{a} \frac{S_v}{S} \frac{1}{\alpha} \frac{v}{U} e_v$	0
Vertical Modes	0	0	✓
Side Bending Modes	0	✓	0

TABLE II

BREAKDOWN OF THE 3-D GUST RESPONSE PROBLEM TO ESSENTIAL COMPONENTS

	Longitudinal Response	Lateral Response
INPUT	w only: Uniform spanwise for moderate size aircraft; Nonuniform spanwise for very large aircraft, and if L is found small	v only (v and w if roll is included)
AIRPLANE	Degrees of freedom: z, vertical motion θ, pitch $a_1, a_2, \ldots$, flexible modes Aerodynamics: Establish aerodynamics by lifting surface theory (or appropriate substitution), including wing induction and tail downwash effects	Degrees of freedom: y, side motion ϕ, roll (ignore for load studies) ψ, yaw b_1, fuselage side bending mode Aerodynamics: Slender body for fuselage Appropriate tail theory
SIMPLIFIED TREATMENT	z only (but use flexible body slope of the lift curve for wings with sweep)	ψ only
OBTAIN (SEPARATELY)	Wing Loads	Vertical tail loads for conventional tails
OBTAIN (COMBINED)	Combine results as indicated in the text to obtain fuselage loads and loads for T-tails	

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